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ABSTRACT

of the dissertation for the scientific degree of Doctor of Philosopy

STRESS-STRAIN STATE ANALYSIS OF A RADIAL INHOMOGENEOUS CYLINDRICAL SHELL

Specialty: 2002.01- Mechanics of deformable solid

Field of science: Mathematics

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GENERAL CHARACTERISTICS OF THE WORK

Rationale and development degree of the topic. The shell theory is a field of science that studies stress-strain state of bodies whose one size called thickness is rather small than the other two sizes of deformable solid mechanics.

Smallness of shell thickness with respect to other sizes causes to create various applied theories that simplify analysis of shells. The study of inhomogeneous, in particular of multi-layer shells is of great importance in shell theory. The construction of applied theories for multi-layer shells consists of 2 two different directions by accepting kinematic hypothesis for one layer or for the packet of layer respectively and obtaining two dimensional equations. Definition of domain of definition of the existing various applied theories for shells, creation of more exact applied theories for them and inability to solve correctly some problems of shell theory for small thickness elastic bodies on the basis of approximate equations makes necessary to study the stress-strain state of a shell on the basis of elasticity theory equations.

At times when the classic elasticity theory became an independent academic subject of continuum mechanics, one of its main hypotheses was inhomogenecity of the material. Then it became known that assumption of homogenecity of the material ignores some its real features. Though the study of the stress-strain state of inhomogeneous elastic bodies if a complex mathematical problem, this approach takes into account their mechanical, geometrical structures more adequately and causes to formation of new quality effect.

In many cases, in order to study the features of an inhomogeneous material, it is assumed that the an elastic material possesses mechanical features expressed by elementary functions. This enables to use the classic methods in solving elasticity theory problems for inhomogeneous bodies and to build solutions that can be accepted as a reference to solve more complicated problems.

By solving the elasticity theory problems for inhomogeneous bodies, the asymptotic methods play an special place and are performed in two directions. In the first direction the studies are based on the asymptotic integration method consisting of superposition of three iteration processes of elasticity theory equations. The basis of this direction was executed in the works of A.L.Goldenweiser, M.I.Huseynzadeh, A.M.Kolos, N.M.Rogachova and other researches. The second direction is based on the method of homogeneous solutions and has found its solution in the works of I.I.Vorovich, O.S.Malkina, N.N.Buzarenko, Y.U.Ustinov, M.K.Mekhtiyev, M.A.Shlenev and others.

Object and subject of the research. Application of asymptotic and numerical methods the stress-strain state of a small thickness inhomogeneous cylindrical shell.

Goal and objectives of the research. Study of stress-strain state of a result inhomogeneous cylindrical shell whose elasticity module change with respect to radius when different boundary conditions are given in a lateral surface on the basis of elasticity theory equations; study of torsional problem of a cylindrical shell whose elasticity module are radius-depended arbitrary positive continuous functions; to build exact and asymptotic solutions of torsional problems of a radial inhomogeneous cylindrical shell whose elastic module change by linear law with respect to the radius; study of elastic wave propagation problem in two-layer and three-layer cylinders by applying numerical-analytic methods.

Research methods. The technique of the study is based on the method of asymptotic integration of elasticity theory equations, homogeneous solutions, discrete orthoqonalization methods.

The main theses to be defended.

- Studying boundary value problems of elasticity theory for a radial, inhomogeneous, small thickness cylindrical shell.
- Defining asymptotic formulas characterizing the stress-strain state of a radial, inhomogeneous, small radius cylindrical shell.
- Studying torsional vibration a radial, inhomogeneous small thickness cylindrical shell.
- Analyzing elastic wave propagation in radial, two-layer and three-layer cylinders.

Scientific novelty of the study. The results obtained in the dissertation work are the followings:

- The stress-strain state of a radical inhomogeneous cylindrical shell was studied on the basis of elasticity theory equations, homogeneous and inhomogeneous solutions were constructed.
- When the lateral surface of the cylindrical shell is free from stress, the determined solutions were classified and it was shown that the first layer consists of the sum of the expanded, simple boundary effect character, boundary layer character solutions. Asymptotic formulas to calculate the stress-strain state of ma cylindrical shell were obtained.
- When homogeneous mixed conditions are given on the lateral surface of a radial inhomogeneous cylindrical shell, it was shown that the homogeneous solution consists of the sum of expanded and boundary layer character solutions.
- It was determined that homogeneous solution for a radial inhomogeneous cylindrical shell with a fixed lateral surface consists of only a boundary layer character solution.
- A problem of torsion of a cylindrical shell whose elastic module are radius dependent arbitrary positive continuous functions, was studied when the lateral surface of a cylindrical shell is free from stress, it was shown that the homogeneous solution consists of the sum of an expanded and boundary layer character solutions. It was obtained that for a cylindrical shell with a fixed lateral surface the torsion problem has only a boundary layer character solution.
- When the lateral surface of a radial inhomogeneous cylindrical shell is free from stress, the torsional vibration problems were studied, exact and asymptotic solutions were built.
- The problems of propagation of asymptotic with respect to the axis symmetric with respect to the axis and torsional elastic waves in a radial two-layer and three-layer cylinder was studied by joint application the numerical analytic methods. The dispersion curves were constructed and their possible asymptotic were determined.

Theoretical and practical importance of the study. This work is of theoretical importance. A new class of solutions that can not be described by applied theories of shells. The obtained asymptotic formulas allow to calculate the stress-strain state of a radial inhomogeneous cylindrical shell to estimate the domains of application of various applied theories existing for a cylindrical shell and to build more exact applied theories.

Approbation and application of the work.

The results of the dissertation work were reported at the I International science and engineering conference, at Baku Engineering University (Baku 2018), at the XXXIX International Scientific practical conference "Advances in Science and Technology" (Moscow 2021), at the conference "Applied problems of mathematics and new information technology" (Sumgayit 2021). The results of the dissertation work were published in scientific journals in the form 6 papers and 3 conference materials.

Author's personal contribution. Except the problem statement, all the results of the study belong to the author.

Author's publications. 6 papers and 3 conference materials in the editions recommended by the Higher Attestation Commission at the President of the Republic of Azerbaijan.

The name of the organization where the work was performed. The work was performed at the department of "General technical disciplines and technology" of the Ganja State University.

Structure and volume of the dissertation (in signs indicating the volume of each structural subdivision separately).

The dissertation work consists of introduction, 3 chapters, conclusion and a list of references,129 pages. The total volume of the work 215871 signs (title page 397 signs, content 1656 signs, introduction 23837 signs, chapter I - 82000 signs, chapter II - 64000 signs, chapter III -42000 signs, conclusion 1981 signs). The dissertation work contains 12 figure and a list of references with 90 names.

The main content of the work

Chapter I is called “A symmetric problem with respect to the axis of the elasticity theory for a radial inhomogeneous cylindrical shell”. In chapter I asymptotic theory pof a aradial inhomogeneous cylindrical shell is given.

In **1.1** a symmetric problem with respect to the axis is considered in the r,φ,z cylindric coordinate system for a radial inhomogeneous isotropic cylindrical shell with the volume $\Gamma=\{r\in[r_1;r_2],\ \varphi\in[0;2\pi],\ z\in[-l_0;l_0]\}$. It is assumed that with respect to the radies the elasticity module alternate by the linear law $G(r)=G_* r,\quad \lambda(r)=\lambda_* r$. The expression of balance equations by the companents of the displacement vector in the cylindric coordivate system is as follows:

$$\left(A_0+\partial_1 A_1+\partial_1^2 A_2\right)\bar{u}=\bar{0}, \qquad (1)$$

Here

$$A_0=\begin{Vmatrix}(2G_0+\lambda_0)(\partial^2+\varepsilon\partial)-2G_0\varepsilon^2 & 0\\ 0 & G_0(\partial^2+\varepsilon\partial)\end{Vmatrix},$$

$$A_1=\begin{Vmatrix}0 & e^{\varepsilon\rho}\left(\varepsilon(G_0+\lambda_0)\partial+\varepsilon^2\lambda_0\right)\\ e^{\varepsilon\rho}\left(\varepsilon^2(2G_0+\lambda_0)+(G_0+\lambda_0)\varepsilon\partial\right) & 0\end{Vmatrix},$$

$$A_2=\begin{Vmatrix}\varepsilon^2 G_0 e^{2\varepsilon\rho} & 0\\ 0 & (2G_0+\lambda_0)\varepsilon^2 e^{2\varepsilon\rho}\end{Vmatrix},$$

$$\bar{u}=(u_\rho;u_\xi)^T,\ \partial_1=\frac{\partial}{\partial\xi},\ \partial_1^2=\frac{\partial^2}{\partial\xi^2},\ \partial=\frac{\partial}{\partial\rho};\ \rho=\frac{1}{\varepsilon}\ln\left(\frac{r}{r_0}\right),\quad \xi=\frac{z}{r_0}$$

are pure variables, $\varepsilon=\frac{1}{2}\ln\left(\frac{r_2}{r_1}\right)$ is a small parameter characterising the thickness of the cylindrical shell, $r_0=\sqrt{r_1 r_2}$, $\rho\in[-1;1]$, $\xi\in[-l;l]$, $l=\frac{l_0}{r_0}$.

It is assumed that the boundary conditions

$$\bar{\sigma} = \left(E_0 + \partial_1 E_1\right)\bar{u}\Big|_{\rho=\pm 1} = \bar{b}^{\pm}(\xi). \tag{2}$$

$$E_0 = \left\| \begin{matrix} \frac{\left(2G_0 + \lambda_0\right)}{\varepsilon}\partial + \lambda_0 & 0 \\ 0 & \frac{G_0}{\varepsilon}\partial \end{matrix} \right\|, E_1 = \left\| \begin{matrix} 0 & \lambda_0 e^{\varepsilon\rho} \\ G_0 e^{\varepsilon\rho} & 0 \end{matrix} \right\|,$$

$$\bar{\sigma} = \left(\sigma_{\rho\rho}, \sigma_{\rho\xi}\right)^T, \ \bar{b}^{\pm}(\xi) = \left(t^{\pm}(\xi);\ d^{\pm}(\xi)\right)^T$$

on the lateral surface if the cylindrical shell, arbitrary boundary conditions retaining it in equilibrium are given on the seat.

The functions $t^{\pm}(\xi)$, $d^{\pm}(\xi)$ in (2) $O(1)$-th order rather smooth functions with respect to ε.

Assuming the parameter ε rather small, according to the first iterative process a special solution called an inhomogeneous slution of (1) satisfying the boundary condition (2) is built according to the asymptotic integration method. The Mitchell-Almansi problem for an inhomogeneous cylindrical shell is studied.

In **1.2** all the solutions of the balance equations (1) satisfying the condition

$$\bar{\sigma} = \left(E_0 + \partial_1 E_1\right)\bar{u}\Big|_{\rho=\pm 1} = \bar{0} \tag{3}$$

where the lateral surface of the cylindrical shell is free from stress, are built.

The solution of the boundary value problem (1), (3) is sought in the from of

$$u_\rho(\rho;\xi) = u(\rho)e^{\alpha\xi}, \quad u_\xi(\rho;\xi) = w(\rho)e^{\alpha\xi} \tag{4}$$

Substituting (4) in (1), (3) we obtain the problem

$$\begin{cases} \left(A_0 + \alpha A_1 + \alpha^2 A_2\right)\bar{a} = \bar{0}, & (5) \\ \left(E_0 + \alpha E_1\right)\bar{a}\Big|_{\rho=\pm 1} = \bar{0} & (6) \end{cases}$$

Here $\bar{a}(\rho) = \left(u(\rho);\ w(\rho)\right)^T$.

As $\varepsilon \to 0$ as a result of analysis by using the asymptotic integration method consisting of three iterative processes in the spectral problem (5), (6) the following solutions are determined:

1) $$u_\rho^{(1)} = -\frac{\lambda_0}{2(G_0+\lambda_0)} C e^{\varepsilon\rho}, \qquad u_\xi^{(1)} = C\xi, \tag{7}$$

$$\sigma_{\rho\rho}^{(1)} = \sigma_{\varphi\varphi}^{(1)} = \sigma_{\rho\xi}^{(1)} = 0, \qquad \sigma_{\xi\xi}^{(1)} = \frac{G_0(2G_0+3\lambda_0)}{G_0+\lambda_0} C e^{\varepsilon\rho}. \tag{8}$$

The solution (7) corresponds to the double eigen value $\alpha = 0$ of the spectral problem (5), (6). The solution (7) determines the stretching of the cylindrical shell along the axis.

2) $$\alpha_j = \varepsilon^{-\frac{1}{2}}(\alpha_{0j} + \varepsilon\alpha_{1j} + \ldots).$$

$$u_\rho^{(2)}(\rho;\xi) = \sum_{j=1}^{4} D_j U_{\rho j}^{(2)}(\rho;\xi), \tag{9}$$

$$u_\xi^{(2)}(\rho;\xi) = \varepsilon^{\frac{1}{2}} \sum_{j=1}^{4} D_j U_{\xi j}^{(2)}(\rho;\xi), \tag{10}$$

here

$$U_{\rho j}^{(2)}(\rho;\xi) = (G_0+\lambda_0)\left\{2 + \varepsilon e_1\left[\alpha_{0j}^2\rho^2 + \left(\frac{2e_1}{1+e_1}\alpha_{0j}^2 - 2\right)\rho\right] + O(\varepsilon^2)\right\}\times$$
$$\times \exp\left(\frac{1}{\sqrt{\varepsilon}}(\alpha_{0j} + \varepsilon\alpha_{1j} + \cdots)\xi\right),$$

$$U_{\xi j}^{(2)}(\rho;\xi) = (G_0+\lambda_0)\left\{-\alpha_{0j}\left(2\rho + \frac{2e_1}{1+e_1}\right) + \varepsilon\left[\frac{(2+3e_1)}{3}\alpha_{0j}^3\rho^3 + \left(\alpha_{0j}e_3 - \right.\right.\right.$$
$$\left.- (1+3e_1)\alpha_{0j}^2 - \frac{e_1^2}{1+e_1}\alpha_{0j}^3\right)\rho^2 + \left(2e_3\alpha_{0j} - 2\alpha_{1j} - 4e_1\alpha_{0j} - 2(1+e_1)\alpha_{0j}^3\right)\rho\Big] +$$
$$\left. + O(\varepsilon^2)\right\}\exp\left(\frac{1}{\sqrt{\varepsilon}}(\alpha_{0j} + \varepsilon\alpha_{1j} + \cdots)\xi\right),$$

$$e_1 = \frac{\lambda_0}{2G_0+\lambda_0}, \quad e_2 = \frac{4G_0(G_0+\lambda_0)}{2G_0+\lambda_0}, \quad e_3 = \frac{2G_0\lambda_0}{2G_0+\lambda_0},$$

α_{0j} are the roots of the biquadratic equation

$$\alpha_{0j}^4 - \frac{3e_1^2}{(1+e_1)^2}\alpha_{0j}^2 + 3 = 0$$

3)

a) $$\alpha_k = \varepsilon^{-1}\left(\beta_{0k} + \varepsilon\beta_{1k} + \ldots\right).$$

$$u_\rho^{(3;1)}(\rho;\xi) = \varepsilon\sum_{k=1}^{\infty} T_k U_{\rho k}^{(3;1)}(\rho;\xi), \tag{11}$$

$$u_\rho^{(3;1)}(\rho;\xi) = \varepsilon\sum_{k=1}^{\infty} T_k U_{\xi k}^{(3;1)}(\rho;\xi), \tag{12}$$

here

$$U_{\rho k}^{(3;1)}(\rho;\xi) = \left[\left(2\beta_{0k}^2\cos\beta_{0k} + \frac{4}{1+e_1}\beta_{0k}\sin\beta_{0k}\right)\sin(\beta_{0k}\rho) - \right.$$

$$\left. - 2\beta_{0k}^2\rho\sin\beta_{0k}\cos(\beta_{0k}\rho) + O(\varepsilon)\right]\exp\left(\frac{1}{\varepsilon}(\beta_{0k} + \varepsilon\beta_{1k} + \cdots)\xi\right),$$

$$U_{\xi k}^{(3;1)}(\rho;\xi) = \left[\left(\frac{2(1-e_1)}{1+e_1}\beta_{0k}\sin\beta_{0k} - 2\beta_{0k}^2\cos\beta_{0k}\right)\cos(\beta_{0k}\rho) - \right.$$

$$\left. - 2\beta_{0k}^2\rho\sin\beta_{0k}\sin(\beta_{0k}\rho) + O(\varepsilon)\right]\exp\left(\frac{1}{\varepsilon}(\beta_{0k} + \varepsilon\beta_{1k} + \cdots)\xi\right),$$

β_{0k} are the roots of the equation

$$\sin 2\beta_{0k} + 2\beta_{0k} = 0,$$

b) $$\alpha_i = \varepsilon^{-1}\left(\beta_{0i} + \varepsilon\beta_{1i} + \ldots\right).$$

$$u_\rho^{(3;2)}(\rho;\xi) = \varepsilon\sum_{i=1}^{\infty} F_i U_{\rho i}^{(3;2)}(\rho;\xi), \tag{13}$$

$$u_\xi^{(3;2)}(\rho;\xi) = \varepsilon\sum_{i=1}^{\infty} F_i U_{\xi i}^{(3;2)}(\rho;\xi)\ , \tag{14}$$

here

$$U_{\rho i}^{(3;2)}(\rho;\xi) = \left[\left(2\beta_{0i}^2\sin\beta_{0i} - \frac{4}{1+e_1}\beta_{0i}\cos\beta_{0i}\right)\cos(\beta_{0i}\rho) - \right.$$

$$-2\beta_{0i}^2\rho\cos\beta_{0i}\sin(\beta_{0i}\rho)+O(\varepsilon)\Big]\exp\left(\frac{1}{\varepsilon}(\beta_{0i}+\varepsilon\beta_{1i}+\cdots)\xi\right),$$

$$U_{\xi i}^{(3;2)}(\rho;\xi)=\left[2\beta_{0i}^2\rho\cos\beta_{0i}\cos(\beta_{0i}\rho)+\left(\frac{2(1-e_1)}{1+e_1}\beta_{0i}\cos\beta_{0i}+2\beta_{0i}^2\sin\beta_{0i}\right)\times\right.$$

$$\times\sin(\beta_{0i}\rho)+O(\varepsilon)\Big]\exp\left(\frac{1}{\varepsilon}(\beta_{0i}+\varepsilon\beta_{1i}+\cdots)\xi\right),$$

β_{0i} are the roots of the equation

$$\sin 2\beta_{0i}-2\beta_{0i}=0.$$

The sum

$$u_\rho(\rho;\xi)=u_\rho^{(1)}+\sum_{j=1}^{4}D_jU_{\rho j}^{(2)}(\rho;\xi)+\varepsilon\sum_{k=1}^{\infty}T_kU_{\rho k}^{(3;1)}(\rho;\xi)+$$

$$+\varepsilon\sum_{i=1}^{\infty}F_iU_{\rho i}^{(3;2)}(\rho;\xi), \qquad (15)$$

$$u_\xi(\rho;\xi)=u_\xi^{(1)}+\sum_{j=1}^{4}D_jU_{\xi j}^{(2)}(\rho;\xi)+\varepsilon\sum_{k=1}^{\infty}T_kU_{\xi k}^{(3;1)}(\rho;\xi)+$$

$$+\varepsilon\sum_{i=1}^{\infty}F_iU_{\xi i}^{(3;2)}(\rho;\xi) \qquad (16)$$

of the solutions (7), (9)- (14) is the total solution of the problem (1), (3).

In **1.3** homogeneous solutions (7), (9)-(14) determined from the asymptotic integration process are classified.

The solution (7) determined in the first iterative process is an expanded solution. The stress state (8) determined by the solution (7) is equivalent to the principal vector P of forces on the arbitrary section $\xi=const$ of the cylindrical shell:

$$P=\frac{4\pi G_0(2G_0+3\lambda_0)}{3(G_0+\lambda_0)}sh(3\varepsilon)C. \qquad (17)$$

The stress state corresponding to the secind and third iterative processes is self-balanced in the section $\xi=const$.

The stress-strain state corresponding to the solution (9), (10) determined by the second iterative process determines the boundary effect in applied theory of shells. The solutions (7), (9),(10) that corresponding to the first and second iteration process, determines the internal stress-strain state of the cylindrical shell.

The solutions (11)- (14) determined by the third iterative process are of boundary character. Boundary layer character layers were not boundary in any applied theories of shells. The first term of the expansion of these solutions with respect to the parameter ε is equivalent to the Saint Venant boundary effect for inhomogeneous plates.

The stress corresponding to the solution determined by the second and third iterative processes are localized on the seat of the cylindrical shell and damp exponentially from its seat to the inside of the shell.

According to (15), (16) the stress-strain state of an inhomogeneous cylindrical shell consists of the sum of the expanded, simple boundary effect and boundary layer character solutions.

In **1.4** the satisfaction of boundary conditions on the seats pof the cylindrical shell is considered

$$\sigma_{\rho\xi}\Big|_{\xi=\pm l} = f_{1s}(\rho), \quad \sigma_{\xi\xi}\Big|_{\xi=\pm l} = f_{2s}(\rho). \qquad (18)$$

It is assumed that the boundary conditions (18) are given on the seats of the cylindrical shell. The functions $f_{1s}(\rho),\, f_{2s}(\rho), (s=1;2)$ contained in (18) are smooth functions satisfying the equilibrium conditions and of $O(1)$ order with respect to the small parameter ε.

According to (17) the constant C is determined by the equality

$$C = \frac{3(G_0+\lambda_0)P}{4\pi G_0(2G_0+3\lambda_0)sh(3\varepsilon)} \; . \qquad (19)$$

The constants D_j, T_k, F_i contained in (9)-(14) are determined from the boundary conditions (18) by using the Lagrange variation principle as follows:

$$\sum_{j=1}^{4} m_{kj} D_{j0} = \tau_k, \quad \left(k=\overline{1,4}\right) \tag{20}$$

$$\sum_{k=1}^{\infty} M_{jk}^{(1)} T_{k0} = d_{0j}^{(1)}, \quad \left(j=1,2,\dots\right) \tag{21}$$

$$\sum_{i=1}^{\infty} Q_{ji}^{(1)} F_{i0} = d_{0j}^{(2)} \quad \left(j=1,2,\dots\right) \tag{22}$$

here

$$m_{kj} = \frac{1}{3\left(2G_0+\lambda_0\right)}\Big[-32G_0\left(G_0+\lambda_0\right)^3\alpha_{0j}^3 + 8G_0\left(G_0+\lambda_0\right)\times$$

$$\times\left(4G_0^2+8G_0\lambda_0+7\lambda_0^2\right)\alpha_{0j}^2\alpha_{0k} - 24G_0\lambda_0^2\left(G_0+\lambda_0\right)\alpha_{0k}\Big]\times$$

$$\times\left(\exp\left(-\frac{\left(\alpha_{0j}+\alpha_{0k}\right)l}{\sqrt{\varepsilon}}\right)+\exp\left(\frac{\left(\alpha_{0j}+\alpha_{0k}\right)l}{\sqrt{\varepsilon}}\right)\right),$$

$$\tau_k = \int_{-1}^{1}\left\{\left[2\left(G_0+\lambda_0\right)\gamma_1 - \alpha_{0k}\left(2\left(G_0+\lambda_0\right)\rho+\lambda_0\right)f_{21}\right]\exp\left(-\frac{\alpha_{0k}l}{\sqrt{\varepsilon}}\right)+\right.$$

$$\left.+\left[2\left(G_0+\lambda_0\right)\gamma_2 - \alpha_{0k}\left(2\left(G_0+\lambda_0\right)\rho+\lambda_0\right)f_{22}\right]\exp\left(\frac{\alpha_{0k}l}{\sqrt{\varepsilon}}\right)\right\}d\rho,$$

$$M_{jk}^{(1)} = \int_{-1}^{1} 4G_0\beta_{0k}^2\left\{\beta_{0k}\left[\cos\beta_{0k}\sin\left(\beta_{0k}\rho\right)-\rho\sin\beta_{0k}\cos\left(\beta_{0k}\rho\right)\right]\cdot\right.$$

$$\cdot\left[\left(2\beta_{0j}\cos\beta_{0j}+\frac{2\left(2G_0+\lambda_0\right)}{G_0+\lambda_0}\sin\beta_{0j}\right)\sin\left(\beta_{0j}\rho\right)-2\beta_{0j}\rho\sin\beta_{0j}\cos\left(\beta_{0j}\rho\right)\right]+$$

$$+\left[\sin\beta_{0k}\cos\left(\beta_{0k}\rho\right)-\beta_{0k}\rho\sin\beta_{0k}\sin\left(\beta_{0k}\rho\right)-\beta_{0k}\cos\beta_{0k}\cos\left(\beta_{0k}\rho\right)\right]\times$$

$$\left.\cdot\left[\left(\frac{2G_0}{G_0+\lambda_0}\sin\beta_{0j}-2\beta_{0j}\cos\beta_{0j}\right)\cos\left(\beta_{0j}\rho\right)-2\beta_{0j}\rho\sin\beta_{0j}\sin\left(\beta_{0j}\rho\right)\right]\right\}d\rho\times$$

$$\times\left[\exp\left(-\frac{\left(\beta_{0k}+\beta_{0j}\right)l}{\varepsilon}\right)+\exp\left(\frac{\left(\beta_{0k}+\beta_{0j}\right)l}{\varepsilon}\right)\right],$$

$$d_{0j}^{(1)} = \int_{-1}^{1}\sum_{s=1}^{2}\left\{ f_{1s}(\rho)\left[\left(2\beta_{0j}\cos\beta_{0j} + \frac{2(2G_0+\lambda_0)}{G_0+\lambda_0}\sin\beta_{0j}\right)\sin(\beta_{0j}\rho) - \right.\right.$$

$$-2\beta_{0j}\rho\sin\beta_{0j}\cos(\beta_{0j}\rho)\Big] + f_{2s}^{*}(\rho)\left[\left(\frac{2G_0}{G_0+\lambda_0}\sin\beta_{0j} - 2\beta_{0j}\cos\beta_{0j}\right)\right.$$

$$\left.\left.\times\cos(\beta_{0j}\rho) - 2\beta_{0j}\rho\sin\beta_{0j}\sin(\beta_{0j}\rho)\right]\right\}d\rho\exp\left((-1)^{s}\frac{\beta_{0j}l}{\varepsilon}\right),$$

$$Q_{ji}^{(1)} = \int_{-1}^{1}4G_0\beta_{0i}^{2}\{\beta_{0i}[\sin\beta_{0i}\cos(\beta_{0i}\rho) - \rho\cos\beta_{0i}\sin(\beta_{0i}\rho)]\cdot$$

$$\cdot\left[\left(2\beta_{0j}\sin\beta_{0j} - \frac{2(2G_0+\lambda_0)}{G_0+\lambda_0}\cos\beta_{0j}\right)\cos(\beta_{0j}\rho) - 2\beta_{0j}\rho\times\right.$$

$$\times\cos\beta_{0j}\sin(\beta_{0j}\rho)\Big] + [\beta_{0i}\sin\beta_{0i}\sin(\beta_{0i}\rho) + \cos\beta_{0i}\sin(\beta_{0i}\rho) +$$

$$+\beta_{0i}\rho\cos\beta_{0i}\cos(\beta_{0i}\rho)]\left[2\beta_{0j}\rho\cos\beta_{0j}\cos(\beta_{0j}\rho) + \left(\frac{2G_0}{G_0+\lambda_0}\cos\beta_{0j} + \right.\right.$$

$$\left.\left.+2\beta_{0j}\sin\beta_{0j}\right)\sin(\beta_{0j}\rho)\right]\Big\}d\rho\left(\exp\left(-\frac{(\beta_{0i}+\beta_{0j})l}{\varepsilon}\right) + \exp\left(\frac{(\beta_{0i}+\beta_{0j})l}{\varepsilon}\right)\right),$$

$$d_{0j}^{(2)} = \int_{-1}^{1}\sum_{s=1}^{2}\left\{ f_{1s}(\rho)\left[\left(2\beta_{0j}\sin\beta_{0j} - \frac{2(2G_0+\lambda_0)}{G_0+\lambda_0}\cos\beta_{0j}\right)\cos(\beta_{0j}\rho) - \right.\right.$$

$$-2\beta_{0j}\rho\cos\beta_{0j}\sin(\beta_{0j}\rho)\Big] + f_{2s}^{*}(\rho)\Big[2\beta_{0j}\rho\cos\beta_{0j}\cos(\beta_{0j}\rho) +$$

$$\left.\left.+\left(\frac{2G_0}{G_0+\lambda_0}\cos\beta_{0j} + 2\beta_{0j}\sin\beta_{0j}\right)\sin(\beta_{0j}\rho)\right]\right]\Big\}d\rho\exp\left((-1)^{s}\frac{\beta_{0j}l}{\varepsilon}\right),$$

$$\gamma_s = \lim_{\varepsilon\to 0}\frac{f_{1s}^{(1)}}{\sqrt{\varepsilon}},\ f_{1s}^{(1)} = \frac{1}{2}\int_{-1}^{1}f_{1s}(\rho)d\rho,\ f_{1s}^{(2)} = f_{1s}(\rho) - f_{1s}^{(1)}.$$

$$f_{2s}^{*}(\rho) = f_{2s}(\rho) - \frac{3Pe^{\varepsilon\rho}}{4\pi sh(3\varepsilon)}.\ (s = 1;2).$$

Here

$$D_j = D_{j0} + \varepsilon D_{j1} + \cdots,\ T_k = T_{k0} + \varepsilon T_{k1} + \cdots,\ F_i = F_{i0} + \varepsilon F_{i1} + \cdots.$$

The matrices of the systems of linear algebraic equations obtained for defining the constants $D_{jp}, T_{kp}, F_{ip}\ (p = 1,2,\ldots)$ are the same with the matrices of the systems (20)- (22). The systems of infinite linear equations (21), (22) are solvable if the expressions located in their right hand side satisfy certain conditions.

In **1.5** we study an elasticity theory problem for an inhomogeneous cylindrical shell with fixed lateral surface. It is assumed that the boundary conditions refaining it in equilibrium state are given on the seats the cylindrical shell. It is shown that the considered problem has the following boundary layer character solutions localized on the seats of the shell and damping exponentially to the inside of the domain:

a)

$$u_\rho^{(3;1)}(\rho;\xi) = \varepsilon \sum_{k=1}^{\infty} T_k \left[\left(\beta_{0k} \sin\beta_{0k} - \frac{3G_0 + \lambda_0}{G_0 + \lambda_0} \cos\beta_{0k} \right) \sin(\beta_{0k}\rho) + \right.$$

$$\left. + \beta_{0k}\rho \cos\beta_{0k} \cos(\beta_{0k}\rho) + O(\varepsilon) \right] \exp\left(\frac{1}{\varepsilon}(\beta_{0k} + \varepsilon\beta_{1k} + \ldots)\xi \right), \tag{23}$$

$$u_\xi^{(3;1)}(\rho;\xi) = \varepsilon \sum_{k=1}^{\infty} T_k \left[\beta_{0k}(\rho\cos\beta_{0k}\sin(\beta_{0k}\rho) - \sin\beta_{0k}\cos(\beta_{0k}\rho)) + O(\varepsilon) \right] \times$$

$$\times \exp\left(\frac{1}{\varepsilon}(\beta_{0k} + \varepsilon\beta_{1k} + \ldots)\xi \right), \tag{24}$$

here the β_{0k} are the roots of the equation

$$\sin 2\beta_{0k} - \frac{2(G_0 + \lambda_0)}{3G_0 + \lambda_0}\beta_{0k} = 0 .$$

b)

$$u_\rho^{(3;2)}(\rho;\xi) = -\varepsilon \sum_{i=1}^{\infty} F_i \left[\left(\frac{3G_0 + \lambda_0}{G_0 + \lambda_0} \sin\beta_{0i} + \beta_{0i}\cos\beta_{0i} \right) \cos(\beta_{0i}\rho) + \right.$$

$$\left. + \beta_{0i}\rho \sin\beta_{0i} \sin(\beta_{0i}\rho) + O(\varepsilon) \right] \exp\left(\frac{1}{\varepsilon}(\beta_{0i} + \varepsilon\beta_{1i} + \ldots)\xi \right), \tag{25}$$

$$u_{\xi}^{(3;2)}(\rho;\xi)=\varepsilon\sum_{i=1}^{\infty}F_i\left[-\beta_{0i}\left(\cos\beta_{0i}\sin(\beta_{0i}\rho)-\rho\cos(\beta_{0i}\rho)\sin\beta_{0i}\right)+O(\varepsilon)\right]\times$$

$$\times\exp\left(\frac{1}{\varepsilon}(\beta_{0i}+\varepsilon\beta_{1i}+\ldots)\xi\right), \qquad (26)$$

here β_{0i} are the roots of the equation

$$\sin 2\beta_{0i}+\frac{2(G_0+\lambda_0)}{3G_0+\lambda_0}\beta_{0i}=0\,.$$

The first term of the expansion of the solutions (23)-(26) with respect to the parameter ε is equivalent to the Saint-Venant boundary effect in theory of inhomogeneous plates. The constants contained in (23)-(26) are determined from the boundary conditions given on the seats of the cylindrical shell T_k, F_i ***** .

In **1.6** we study an elasticity theory problem for a small thickness cylindrical shell with homogeneous mixed boundary conditions

$$u_{\rho}\big|_{\rho=\pm1}=0,\;\; \sigma_{\rho\xi}\big|_{\rho=\pm1}=0,$$

given on the lateral surface and with boundary conditions given in the seat and refaining the cylindrical shell in equilibrium state.

As a result of application of the asymptotic integration method as $\varepsilon\to 0$ we determine the following two groups of solutions :

1)
$$u_{\rho}^{(1)}=De_0\left(\frac{sh(k_2-\varepsilon)}{sh(\varepsilon t)}e^{k_1\rho}-\frac{sh(k_1-\varepsilon)}{sh(\varepsilon t)}e^{k_2\rho}-e^{\varepsilon\rho}\right), \qquad (27)$$

$$u_{\xi}^{(1)}=D\xi, \qquad (28)$$

here $k_1=\dfrac{-\varepsilon(1+t)}{2},\quad k_2=\dfrac{\varepsilon(t-1)}{2},\; t=\sqrt{\dfrac{10G_0+\lambda_0}{2G_0+\lambda_0}},$

$$e_0=\frac{\lambda_0}{2(G_0+\lambda_0)}\,. \qquad (29)$$

2)

a)
$$u_{\rho}^{(3;1)}(\rho;\xi)=\varepsilon\sum_{k=1}^{\infty}T_k\left(\beta_{0k}\cos\beta_{0k}\sin(\beta_{0k}\rho)+O(\varepsilon)\right)\times$$

$$\times \exp\left(\frac{1}{\varepsilon}\left(\beta_{0k}+\varepsilon\beta_{1k}+\cdots\right)\xi\right) \tag{30}$$

$$u_{\xi}^{(3;1)}(\rho;\xi) = -\varepsilon\sum_{k=1}^{\infty} T_k\left(\beta_{0k}\cos\beta_{0k}\cos(\beta_{0k}\rho)+O(\varepsilon)\right)\times$$

$$\times \exp\left(\frac{1}{\varepsilon}\left(\beta_{0k}+\varepsilon\beta_{1k}+\cdots\right)\xi\right), \tag{31}$$

here β_{0k} are the solution of the equation

$$\sin\beta_{0k} = 0 .$$

b) $$u_{\rho}^{(3;2)}(\rho;\xi) = \varepsilon\sum_{i=1}^{\infty} F_i\left(\beta_{0i}\sin\beta_{0i}\cos(\beta_{0i}\rho)+O(\varepsilon)\right)\times$$

$$\times \exp\left(\frac{1}{\varepsilon}\left(\beta_{0i}+\varepsilon\beta_{1i}+\cdots\right)\xi\right), \tag{32}$$

$$u_{\xi}^{(3;2)}(\rho;\xi) = \varepsilon\sum_{i=1}^{\infty} F_i\left(\beta_{0i}\sin\beta_{0i}\sin(\beta_{0i}\rho)+O(\varepsilon)\right)\times$$

$$\times \exp\left(\frac{1}{\varepsilon}\left(\beta_{0i}+\varepsilon\beta_{1i}+\cdots\right)\xi\right), \tag{33}$$

here $\beta_{0i}-$ are the solutions of the equation

$$\cos\beta_{0i} = 0 .$$

The solution (27),(28) is an expanded solution and this solution determines the inner stress-strain state of the cylindrical shell. The expanded solution is equivalent to the principal vector of forces acting in the arbitrary upper section $\xi = const$ of the cylindrical shell:

$$P = 2\pi d_0 D, \tag{34}$$

here

$$d_0 = \frac{\varepsilon\lambda_0^2}{2(G_0+\lambda_0)sh(\varepsilon t)}\left(\frac{1-t}{k_1+2\varepsilon}sh(k_2-\varepsilon)sh(k_1+2\varepsilon)-\right.$$

$$\left.-\frac{1+t}{k_2+2\varepsilon}sh(k_1-\varepsilon)sh(k_2+2\varepsilon)\right)+\frac{2G_0(2G_0+\lambda_0)}{3(G_0+\lambda_0)}sh(3\varepsilon).$$

The stress state corresponding to the third iterative process is selfbalanced in arbitrary section $\xi = const$. The solutions (30)-(33) determined by the third iterative process, is of boundary layer character. The stresses corresponding to these solutions are localized in the seat of the cylindrical shell and damp towards the inside of the domain.

The stress-strain state of an inhomogeneous cylindrical shell consists of the sum of the expanded and boundary layer character solutions.

The constant D is determined according to (34). The unknown constants T_k, F_i contained in (30)-(33) arev found from the boundary conditions given in the scats of the cylindrical shell by using the Lagrange variation principle.

When on the lateral surface of the cylindrical shell the homogeneous mixed boundary conditions

$$u_\xi\big|_{\rho=\pm1} = 0, \ \sigma_{\rho\rho}\big|_{\rho=\pm1} = 0,$$

and when on the seats of the cylindrical shell the boundary conditions refaining the cylindrical state in equilibrium stte are given, an elasticity theory problem is studied for a small thickness inhomogeneous cylindrical shell. It is shown that the problem under consideration has a boundary layer character solutions locallized on the seats of the shell and exponentially decaying towards the inside of the domain.

In **1.7** a problem is solved numerically for a radial inhomogeneous and homogeneous cylindrical shell with a fixed lateral surface. The obtained results are compared and its impact of inhomogeneity on the stress strain state is estimated.

Chapter II is called “A torsion problem for a radial inhomogeneous cylindrical shell”. In chapter II asymptotic theory of a torsion problem is given when various boundary conditions are given on the lateral surface of a radial inhomogeneous cylindrical shell.

In **2.1** considers a torsion problem of a cylindrical shell whose elastic module are radius dependent arbitrary continuous functions. The expression of the equilibrium equation describing the torsion of the cylindrical shell in are cylindrical coordinate system r, φ, z by the displacemenet vector components is a sfollows:

$$\frac{\partial}{\partial r}\left[G(r)\left(\frac{\partial u_\varphi}{\partial r}-\frac{u_\varphi}{r}\right)\right]+\frac{2G(r)}{r}\left(\frac{\partial u_\varphi}{\partial r}-\frac{u_\varphi}{r}\right)+G(r)\frac{\partial^2 u_\varphi}{\partial z^2}=0, \quad (35)$$

Assume that the lateral surface of the cylindrical shell is load-free

$$\sigma_{r\varphi}=G(r)\left(\frac{\partial u_\varphi}{\partial r}-\frac{u_\varphi}{r}\right)\Bigg|_{r=r_s}=0, \quad (36)$$

and on the seats of the cylindrical shell the boundary conditions

$$\sigma_{\varphi z}=G(r)\frac{\partial u_\varphi}{\partial z}\Bigg|_{z=\pm l}=f^{\pm}(r), \quad (37)$$

refaining it in equilibrium state are given.

In (37), $f^{\pm}(r)$ are smooth functions satisfying the equilibrium conditions.

The solution of the equation (35) is sought in the form

$$u_\varphi(\rho,\xi)=v(r)m(z), \quad (38)$$

Here the functio $m(z)$ satisfies the condition

$$m''(z)-\mu^2 m(z)=0. \quad (39)$$

Substituting (38) in (35),(36) according to (39) we obtain the boundary value problem

$$\left[G(r)\left(v'(r)-\frac{v(r)}{r}\right)\right]'+\frac{2G(r)}{r}\left(v'(r)-\frac{v(r)}{r}\right)+$$

$$+\mu^2 G(r)v(r)=0, \quad (40)$$

$$G(r)\left(v'(r)-\frac{v(r)}{r}\right)\Bigg|_{r=r_s}=0, \quad (41)$$

$(s=1;2)$.

The solution of the boundary value problem (40), (41) is determined by the equality

$$u_\varphi(r,z)=A_0rz+\sum_{k=1}^{\infty}v_k(r)\left(A_{1k}e^{-\mu_k z}+B_{1k}e^{\mu_k z}\right), \tag{42}$$

Here A_{1k}, B_{1k} are arbitrary constants. For the torgue $M_{bur.}$ of stresses acting on the section $z=const$ the equality

$$M_{bur.}=2\pi A_0\int_{r_1}^{r_2}G(r)r^3dr. \tag{43}$$

is valid.

The $u_{\varphi 0}(r,z)=A_0rz$ is an expanded solution and this solution determines the inner stress-strain state of the cylindrical shell. According (43) the constant is equivalent to the torgue $M_{bur.}$ of stresses acting in the section $z=const$: A_0 :

$$A_0=\frac{M_{bur.}}{2\pi\int_{r_1}^{r_2}G(r)r^2dr}$$

Boundary layer character solution

$$\sum_{k=1}^{\infty}v_k(r)\left(A_{1k}e^{-\mu_k z}+B_{1k}e^{\mu_k z}\right) \tag{44}$$

is localized on the seats of the cylindrical shell and damps towards the inside of the domain.

The general solution of the problem consists of the sum of the expanded solution and a boundary layer character solution.

A problem on satisfaction of boundary conditions (37) given on the seats of the cylindrical shell, is considered and arbitrary constants A_{1k}, B_{1k} are determined.

In **2.2** a problem on torsion of a radial inhomogeneous cylindrical shell whose elastic module are radius dependent arbitrary continuous functions, with fixed lateral surface and the given stresses on the seats refaining it in equilibrium state, considered. It is shown that the solution is of boundary layer character.

In **2.3** by the inhomogeneous solution method we study a problem of torsion of a radial inhomogeneous cylindrical shell with load-free lateral surface and whose elastic module are dependent force function. The exact solution of the considered problem is built. Satisfying the homogeneous boundary conditions given on the lateral surface, we classify the roots of the obtained characteristics equation and build asymptotic solution corresponding to these roots.

In **2.4** we consider a problem of torsion of a darial inhomogeneous cylindrical shell with a fixed lateral surface and whose elastic module are radius dependent force function by the method of homogeneous solutions. The exact and asymptotic solution of the considered problem is determined.

In **2.5** we study torsional vibrations of a radial inhomogeneous cylindrical shell whose elastic modules and material density changes by the linear law

$$G(r) = G_* r, \ m(r) = m_* r$$

with respect to the radius (G_*, m_* are certain constants).

The expression of the motion equation characterising the torsional vibration component is as follows:

$$\frac{\partial^2 u_\varphi}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial u_\varphi}{\partial \rho} - \frac{2}{\rho^2} u_\varphi + \frac{\partial^2 u_\varphi}{\partial \xi^2} = \frac{m_0}{G_0}\frac{\partial^2 u_\varphi}{\partial \tau^2} \tag{45}$$

Here $\rho = \frac{r}{r_0}, \xi = \frac{z}{r_0}$ are new pure variables; $m_0 = \frac{m_* r_0}{m_1}$, $G_0 = \frac{G_* r_0}{G_1}$, $\tau = \frac{t}{r_0}\sqrt{\frac{G_1}{m_1}}$ are pure variables; are characteristic variables with density sise m_1 and elastic modulus size G_1; $r_0 = \frac{r_1 + r_2}{2}$ – are mediam surface radii.

It is assumed that the lateral surface of the cylindrical shell is load free:

$$\sigma_{\rho\varphi} = G_0 \rho \left(\frac{\partial u_\varphi}{\partial \rho} - \frac{u_\varphi}{\rho} \right)\Bigg|_{\rho = \rho_s} = 0, \tag{46}$$

and on its seats the boundary conditions

$$\sigma_{\varphi\xi} = G_0 \rho \frac{\partial u_\varphi}{\partial \xi}\bigg|_{\xi=\pm l_0} = f^{\pm}(\rho) e^{i\lambda\tau}, \qquad (47)$$

are given.

Here λ is vibration frequency.

The solution of the equation (45) is sought in the form

$$u_\varphi(\rho, \xi, \tau) = v(\rho) a(\xi) e^{i\lambda\tau} \ . \qquad (48)$$

Here the function $a(\xi)$ is the solution of the equation

$$a''(\xi) - \mu^2 a(\xi) = 0 .$$

Substituting (48) in (45), (46) we obtain the boundary value problem

$$v''(\rho) + \frac{2}{\rho} v'(\rho) + \left(\mu^2 + \frac{m_0}{G_0}\lambda^2 - \frac{2}{\rho^2} \right) v(\rho) = 0, \qquad (49)$$

$$G_0 \rho \left(v'(\rho) - \frac{v(\rho)}{\rho} \right)\bigg|_{\rho=\rho_s} = 0. \qquad (50)$$

Writing the solution

$$v(\rho) = \rho^{-\frac{1}{2}} \left(C_1 J_{\frac{3}{2}}(\alpha\rho) + C_2 Y_{\frac{3}{2}}(\alpha\rho) \right). \qquad (51)$$

of the equation (49) in the boundary conditions (50) from the existence of non-trivial solution of the obtained system of linear homogeneous algebraic equations we obtain the dispersion equation

$$\Delta_1(\mu, \lambda, \rho_1, \rho_2) = \frac{2G_0^2}{\pi \rho_1^2 \rho_2^2} \left\{ \left[\alpha^4 \rho_1 \rho_2 + 3\left(3\rho_1\rho_2 - \rho_1^2 - \rho_2^2\right)\alpha^2 + 9 \right] \times \right.$$

$$\left. \times \frac{\sin(\alpha(\rho_1 - \rho_2))}{\alpha^3} + 3(\rho_1 - \rho_2)\left(3 + \alpha^2 \rho_1 \rho_2\right) \frac{\cos(\alpha(\rho_2 - \rho_1))}{\alpha^2} \right\} = 0 \cdot \qquad (52)$$

Here $J_{\frac{3}{2}}(\alpha\rho), Y_{\frac{3}{2}}(\alpha\rho)$ are first and second kind Bessel functions; $\alpha = \left(\mu^2 + \frac{m_0}{G_0}\lambda^2 \right)^{\frac{1}{2}}$.

We introduce a small parameter $\varepsilon = \frac{r_2 - r_1}{2r_0}$ characterizing the thickness of the cylindrical shell. The number $\alpha^2 = 0$ is the root of the dispersion equation (52). According to the equality $\alpha^2 = 0$

$$\mu = \pm i\lambda \sqrt{\frac{m_0}{G_0}} \tag{53}$$

as $\varepsilon \to 0$ for the λ satisfying the condition $\lambda = O(1)$ the dispersion equation has a denumerable number roots

$$\mu_k = \frac{\delta_k}{\varepsilon} + O(\varepsilon), \sin 2\delta_k = 0 \tag{54}$$

as $\lambda \to \infty$ and $\varepsilon\lambda \to const$ ($\lambda = \lambda_0 \varepsilon^{-1}, \varepsilon \to 0$) has the denumerable number roots

$$\mu_k = \frac{\gamma_k}{\varepsilon} + O(\varepsilon), \sin\left(2\sqrt{\gamma_k^2 + \frac{m_0}{G_0}\lambda^2}\right) = 0. \tag{55}$$

Asymptotic expression corresponding to the roots of (53), (54), (55) of dispersion equations of displacement vector and stress tensor components are determined.

In **2.6** considers a torsional vibration of a radial inhomogeneous shell with a fixed lateral surface and whose elasticity modulus and material density changes by the linear law. The exact solution of the problem is built. Asymptotics of the roots of the dispersion equation are determined, asymptotics expressions for displacement and stress tensor components corresponding to these roots are structured.

Chapter III is called "Propogation of elastic waves in radial two-layer and three-layer cylinders". Chapter III propagation of elastic waves in a two layer and three-layer cylinder is studied by joint application of numerical and analytical methods.

In **3.1** a problem on propagation of asymmetric elastic waves with respect to the axis in a radial three-layer cylinder with load-free lateral surface is solver numerically. The dispersion curve $\alpha_t = \alpha_t(\Omega)$ showing the dependence of the wave number α on the frequency Ω is built.

Choosing the stress tensor components $\sigma_{\rho\rho}^{(k)}$, $\sigma_{\rho\xi}^{(k)}$, $\sigma_{\rho\varphi}^{(k)}$ of the "k" number layer, the displacement vector components $u_{\rho}^{(k)}$, $u_{\xi}^{(k)}$, $u_{\varphi}^{(k)}$ as desired vector components from the displacement equations and from the expression of stress tensor components by the displacement vector components we obtain the system of partial equations

$$\frac{\partial\sigma_{\rho\rho}^{(k)}}{\partial\rho}=\frac{1-2\nu_k}{\nu_k-1}\frac{1}{\rho}\sigma_{\rho\rho}^{(k)}-\frac{\partial\sigma_{\rho\xi}^{(k)}}{\partial\xi}-\frac{1}{\rho}\frac{\partial\sigma_{\rho\varphi}^{(k)}}{\partial\varphi}+\frac{2G_k}{1-\nu_k}\frac{1}{\rho^2}u_{\rho}^{(k)}+m_k\frac{\partial^2u_{\rho}^{(k)}}{\partial\tau^2}+$$
$$+\frac{2G_k\nu_k}{1-\nu_k}\frac{1}{\rho}\frac{\partial u_{\xi}^{(k)}}{\partial\xi}+\frac{2G_k}{1-\nu_k}\frac{1}{\rho^2}\frac{\partial u_{\varphi}^{(k)}}{\partial\varphi},$$

$$\frac{\partial\sigma_{\rho\xi}^{(k)}}{\partial\rho}=\frac{\nu_k}{\nu_k-1}\frac{\partial\sigma_{\rho\rho}^{(k)}}{\partial\xi}-\frac{1}{\rho}\sigma_{\rho\xi}^{(k)}-\frac{2G_k\nu_k}{1-\nu_k}\frac{1}{\rho}\frac{\partial u_{\rho}^{(k)}}{\partial\xi}-\frac{2G_k}{1-\nu_k}\frac{\partial^2u_{\xi}^{(k)}}{\partial\xi^2}-$$
$$-G_k\frac{1}{\rho^2}\frac{\partial^2u_{\xi}^{(k)}}{\partial\varphi^2}+m_k\frac{\partial^2u_{\xi}^{(k)}}{\partial\tau^2}-\frac{G_k(1+\nu_k)}{1-\nu_k}\frac{1}{\rho}\frac{\partial^2u_{\varphi}^{(k)}}{\partial\xi\partial\varphi}, \qquad (56)$$

$$\frac{\partial\sigma_{\rho\varphi}^{(k)}}{\partial\rho}=\frac{\nu_k}{\nu_k-1}\frac{1}{\rho}\frac{\partial\sigma_{\rho\rho}^{(k)}}{\partial\varphi}-\frac{2}{\rho}\sigma_{\rho\varphi}^{(k)}-\frac{2G_k}{1-\nu_k}\frac{1}{\rho^2}\frac{\partial u_{\rho}^{(k)}}{\partial\varphi}-\frac{G_k(1+\nu_k)}{1-\nu_k}\frac{1}{\rho}\frac{\partial^2u_{\xi}^{(k)}}{\partial\varphi\partial\xi}-$$
$$-\frac{2G_k}{1-\nu_k}\frac{1}{\rho^2}\frac{\partial^2u_{\varphi}^{(k)}}{\partial\varphi^2}-G_k\frac{\partial^2u_{\varphi}^{(k)}}{\partial\xi^2}+m_k\frac{\partial^2u_{\varphi}^{(k)}}{\partial\tau^2},$$

$$\frac{\partial u_{\rho}^{(k)}}{\partial\rho}=\frac{1-2\nu_k}{2(1-\nu_k)G_k}\sigma_{\rho\rho}^{(k)}-\frac{\nu_k}{1-\nu_k}\frac{u_{\rho}^{(k)}}{\rho}-\frac{\nu_k}{1-\nu_k}\frac{\partial u_{\xi}^{(k)}}{\partial\xi}-\frac{\nu_k}{1-\nu_k}\frac{1}{\rho}\frac{\partial u_{\varphi}^{(k)}}{\partial\varphi},$$

$$\frac{\partial u_{\xi}^{(k)}}{\partial\rho}=\frac{1}{G_k}\sigma_{\rho\xi}^{(k)}-\frac{\partial u_{\rho}^{(k)}}{\partial\xi},$$

$$\frac{\partial u_{\varphi}^{(k)}}{\partial\rho}=\frac{1}{G_k}\sigma_{\rho\varphi}^{(k)}-\frac{1}{\rho}\frac{\partial u_{\rho}^{(k)}}{\partial\rho}+\frac{1}{\rho}u_{\varphi}^{(k)},$$

Here by G_k we denote the shift modulus of the "k" number layer, by ν_k the Poisson ratio, by m_k the material density.

According to the contactness condition of layer composing the cylinder, the equality

$$
\begin{cases}
\sigma_{\rho\rho}^{(s)}(\rho_{2s},\varphi,\xi,\tau)=\sigma_{\rho\rho}^{(s+1)}(\rho_{1s+1},\varphi,\xi,\tau),\\
\sigma_{\rho\xi}^{(s)}(\rho_{2s},\varphi,\xi,\tau)=\sigma_{\rho\xi}^{(s+1)}(\rho_{1s+1},\varphi,\xi,\tau),\\
\sigma_{\rho\varphi}^{(s)}(\rho_{2s},\varphi,\xi,\tau)=\sigma_{\rho\varphi}^{(s+1)}(\rho_{1s+1},\varphi,\xi,\tau),\\
u_{\rho}^{(s)}(\rho_{2s},\varphi,\xi,\tau)=u_{\rho}^{(s+1)}(\rho_{1s+1},\varphi,\xi,\tau),\\
u_{\xi}^{(s)}(\rho_{2s},\varphi,\xi,\tau)=u_{\xi}^{(s+1)}(\rho_{1s+1},\varphi,\xi,\tau),\\
u_{\varphi}^{(s)}(\rho_{2s},\varphi,\xi,\tau)=u_{\varphi}^{(s+1)}(\rho_{1s+1},\varphi,\xi,\tau),
\end{cases}
\tag{57}
$$

According to the fact that the lateral surface of the cylinder is load-free, the equality

$$
\begin{cases}
\sigma_{\rho\rho}^{(1)}(\rho_{11},\varphi,\xi,\tau)=0,\\
\sigma_{\rho\xi}^{(1)}(\rho_{11},\varphi,\xi,\tau)=0,\\
\sigma_{\rho\varphi}^{(1)}(\rho_{11},\varphi,\xi,\tau)=0,\\
\sigma_{\rho\rho}^{(3)}(1,\varphi,\xi,\tau)=0,\\
\sigma_{\rho\xi}^{(3)}(1,\varphi,\xi,\tau)=0,\\
\sigma_{\rho\varphi}^{(3)}(1,\varphi,\xi,\tau)=0.
\end{cases}
\tag{58}
$$

are valid $(s=1,2)$. Here ρ_{1k},ρ_{2k} are internal and external radio of the "k" number layer, respectively.

Looking for the solution of the problem (56)- (58) in the form of

$$
\left(\sigma_{\rho\rho}^{(k)},\ \sigma_{\rho\xi}^{(k)},\ \sigma_{\rho\varphi}^{(k)},\ u_{\rho}^{(k)},\ u_{\xi}^{(k)},\ u_{\varphi}^{(k)}\right)=
$$

$$
=\left(\tilde{\sigma}_{\rho\rho}^{(k)}(\rho)\cos(n\varphi),\ \tilde{\sigma}_{\rho\xi}^{(k)}\cos(n\varphi),\tilde{\sigma}_{\rho\varphi}^{(k)}(\rho)\sin(n\varphi),\tilde{u}_{\rho}^{(k)}(\rho)\cos(n\varphi),\right.
$$

$$
\left.\tilde{u}_{\xi}^{(k)}(\rho)\cos(n\varphi),\tilde{u}_{\varphi}^{(k)}(\rho)\sin(n\varphi)\right)e^{i(\alpha\xi-\Omega\tau)}
$$

as a result, we obtain the boundary value problem

$$\begin{cases} \dfrac{d\overline{y_k}}{d\rho} = A_k(\rho,\Omega,\alpha,n)\overline{y_k}, \\ C\overline{y_1}(\rho_{11}) = \overline{0}, \\ \overline{y_s}(\rho_{2s}) = \overline{y}_{s+1}(\rho_{1s+1}), \\ C\overline{y_3}(1) = \overline{0}. \end{cases} \qquad (59)$$

Here $\bar{y}_k(\rho) = \left(\tilde{\sigma}^{(k)}_{\rho\rho}(\rho), \tilde{\sigma}^{(k)}_{\rho\xi}(\rho), \tilde{\sigma}^{(k)}_{\rho\varphi}(\rho), \tilde{u}^{(k)}_{\rho}(\rho), \tilde{u}^{(k)}_{\xi}(\rho), \tilde{u}^{(k)}_{\varphi}(\rho)\right)^T$ is the desired vector function;

$$A_k(\rho,\Omega,\alpha,n) = \left\| a^{(k)}_{ji} \right\|; \quad i, j = \overline{1,6}. \ k = 1,2,3; \ s = 1,2.$$

Baundary value problem (59) contains the spectral parameter Ω and α. By solving the problem (59) by means of the discrete orthogonalization method, we determine thhhe dispersion dependence $\alpha_t = \alpha_t(\Omega)$ is determined. Real eigen values α_t determine energy carrying homogeneous elastic waves expanded alonh the cylinders axis, the complex and pure omaginary eigen values determine inhomogeneous elastic waves not carrying energy. Since the dispersion curves determine main characteristics of an unbounded cylinders α_t. Since real dispersion curves $\alpha_t = \alpha_t(\Omega)$ define mean characteristics of unbounded cylinder, for $\frac{G_1}{G_2} = \frac{1}{4}$; $\frac{G_3}{G_2} = 4$; $\frac{m_1}{m_2} = \frac{1}{6}$; $\frac{m_3}{m_2} = 6$; $v_1 = v_2 = v_3 = 0{,}3$; $\rho_{11} = 0{,}1$; $\rho_{21} = 0{,}4$; $\rho_{13} = 0{,}7$; $\rho_{23} = 1$ real dispersion curves are built.

In **3.2** we study propagation of symmetric elastic waves with respect to the axis in a radial three-layer infinite cylinder. The dispersion curves are built for radial three-layer cylinder by joint application of asymptotic and numerical methods. Near the point $(\alpha;\Omega) = (0;0)$ for the first dispersion curve the asymptotic expression

$$\alpha = \sqrt{\frac{\sum_{k=1}^{3} m_k\left(\rho_{2k}^2 - \rho_{1k}^2\right)}{\sum_{k=1}^{3} 2G_k\left(\rho_{2k}^2 - \rho_{1k}^2\right)}}\,\Omega + O\left(\Omega^2\right)$$

is determined.

As $\alpha \to \infty$, $\Omega \to \infty$ $\left(\lim \frac{\Omega}{\alpha} = const\right)$ possible asymptotic of dispersion curves are determined.

When the ratio $\frac{G_2}{G_1}$ for a three-layer cylindrical shell whose inner and external layers posses the same elastic properties and middle line consisting of a soft material is a small parameter the initial points $\left(0,\Omega_t\right)$ of the real dispersion curves $\alpha_t = \alpha_t(\Omega)$ are found in the plane (α,Ω).

For $\frac{G_2}{G_1}=0{,}02;\ \frac{m_2}{m_1}=0{,}2;\ \rho_{11}=0{,}2\ \ \rho_{21}=0{,}5;\ \ \rho_{13}=0{,}6;\ \rho_{23}=1$ the dispersion curves are constructed (fig 1).

By increasing Ω from 0 to 1,68 the first dispersion curve originating from the point (0,0) approaches to the straight line whose angle coefficietns equals the propagation velocity of lateral wave in solid layers. Beginning with the value $\Omega=1{,}68$ the asymptotes of the dispersion curve is a ctraightline whose angle coefficient equals the phase velocity of longitudinal wave in a soft layer. By increasing the α for the first dispersion curve the asymptotic will the equal the phase velocity of Rayleigh waves whose angle coefficient expands along the free surface of the cylinder. By increasing Ω the second dispersion curve approaches the straightline whose angle coefficients equals the phase velocity of longitudinal wave in solid layer. In the range $1{,}48<\Omega<2{,}78$ the asymptotic of the dispersion curve is a straightline whose angle coefficient equals the phase velocity of lateral wave in a solid layer. For $\Omega>2{,}78$ by increasing Ω -the dispersion curve approaches the straightline whose angle coefficient equals the velocity of a longitudinal wave in a soft layer.

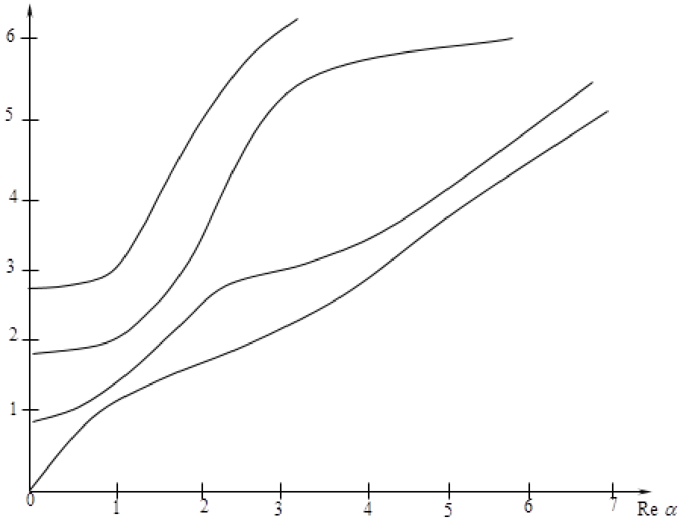

Fig. 1

In **3.3** we consider torsional wave propagation in a radial three-layer infinite cylinder. Near $(\alpha;\Omega)=(0;0)$ we determine the asymptotic expression

$$\alpha = \sqrt{\frac{\sum_{k=1}^{3} m_k\left(\rho_{2k}^4 - \rho_{1k}^4\right)}{\sum_{k=1}^{3} G_k\left(\rho_{2k}^4 - \rho_{1k}^4\right)}}\Omega + O\left(\Omega^2\right)$$

for the first dispersion curve. We determine the initial points $(0,\Omega_t)$ of the real dispersion curves $\alpha_t = \alpha_t(\Omega)$ for a three-layer cylinder whose inner and extermal layers were composed of the material with the same elastic propertiers the middle layer composed of a softs material and for

$$\frac{G_2}{G_1}=0{,}02;\ \frac{m_2}{m_1}=0{,}2;\ \rho_{11}=0{,}2\ \ \rho_{21}=0{,}5;\ \ \rho_{13}=0{,}6;\ \rho_{23}=1$$

dispersion curves are built (fig. 2).

Increasing Ω from 0 to 5,48 the first dispersion curve originating from the point (0,0) approaches to a straight line whose angle coefficients equals the velocity of propagation of lateral wave in a solid layer. Beginning from the value of $\Omega = 5{,}48$ the asymptotic of the dispersion curve is a straight line whose angle coefficients equals the phase velocity of lateral wave in a soft layer. For $\Omega \geq 0{,}865$ there exist two expanded waves in the cylinder. By increasing Ω the dispersion curve approaches the straightline whose angle coefficients equals the phase velocity of lateral wave in solid layer. As $\alpha \to \infty$, $\Omega \to \infty$ $\left(\lim \frac{\Omega}{\alpha} = const \right)$ the asymptotic of dispersion curves is a straightline whose angle coefficients equals $\sqrt{\frac{m_1}{m_2} \frac{G_2}{G_1}}$.

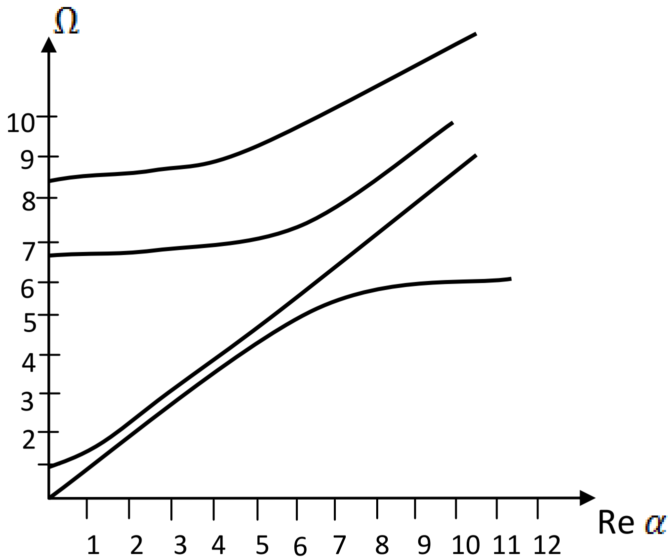

Fig. 2

In **3.4** we consider propagation of symmetric elastic waves with respect to the axis in a radial two-layer cylinder. Dispersion curves are built for $\frac{G_2}{G_1} = 0{,}01;\ \frac{m_2}{m_1} = 0{,}2;\ \rho_{11} = 0{,}99,\ \rho_{12} = 1,\ \rho_{22} = 1{,}03$ in a cylinder with solid inner layer and external soft layer (fig. 3).

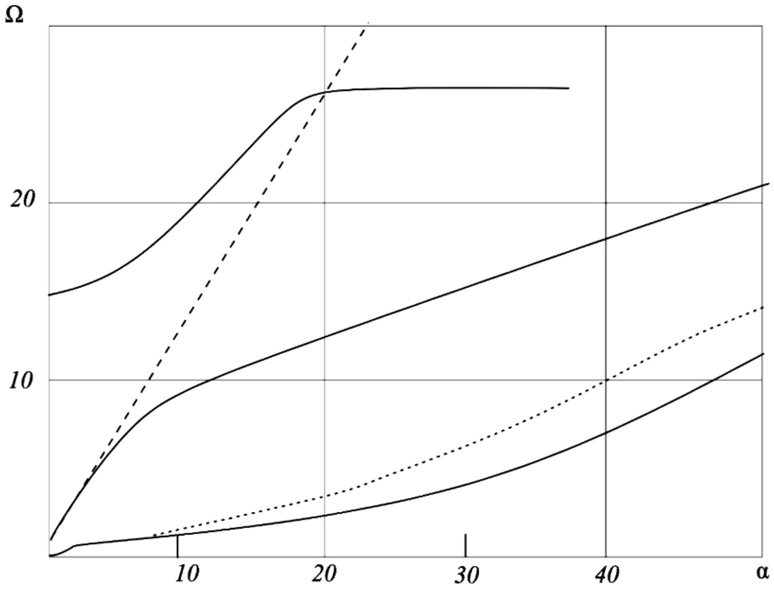

Fig. 3

Beginning with the values $\Omega = 1{,}36$ two waves propage in the cylinder. According to the unigue normal hypothesis, based on the structured applied theory, dispersion curves are built for the problem of elastic waves propagation in a two-layer cylinder (it is shown with broken lines). In the first mode, the results obtained on the basis of applied theory for frequencies satisfying the condition $\Omega \leq 2$ are close to the results obtained from the solution three dimensional problem. In the second mode for the values of the frequency $\Omega \leq 5$ the results obtained from the applied theory slightly differ from the results obtained from the solution of three dimensional problem.

Conclusion

The dissertation work was devoted to the study of the stress-strain state of a small thickness inhomogeneous cylindrical shell on the basis of elasticity theory equations.

- Problems of axial symmetricity of a small thickness cylindrical shell whose elastic module change by the linear law with respect to the radius is studied by applying the asymptotic integration method. Homogeneous and nonhomogeneous solutions were built. The character of the stress-strain state of the cylindrical shell was determined. When the lateral surface of the cylindrical shell is stress free, it is shown that the determined homogeneous solution consists of the sum of expended, simple boundary effect character, boundary layer character solutions. New classes of solutions that can not be determined by the existing applied theories, were built. Asymptotic formulas to calculate the stress-strain state of the cylindrical shell were obtained.
- Given homogeneous mixed boundary conditions of a small thickness cylindrical shell whose elastic module change with respect to the radius by the linear law it was shown that the homogeneous solution consists of the sum of the expands and boundary layer character solutions.
- It was determined that when the lateral surface of a small thickness cylindrical shell whose elastic module with respect to the axis change by the linear law the solution consists of only of boundary layer character solutions.
- Given various boundary conditions on a lateral surface of a radial inhomogeneous cylindrical shell, the torsion problem was studied by asymptotic integration and homogeneous solutions method. It was shown that when the lateral surface of the cylindrical shell is stress free, the homogeneous solution consists of the sum of the expands and boundary layer character solutions and when the lateral surface of the cylindrical shell is fixed, the homogeneous solution is only of a boundary layer character.
- When the lateral surface of a cylindrical shell is free from stresses and when the lateral surface is fixed, the torsional vibrations

of a cylindrical shell were studied. Exact and asymptotic solutions were built. The asymptotic expressions to determine the stress-strain state were determined at various values of frequency.

- The problem of elastic waves propagation in radial two-layer and three-layer cylinder was studying by joint application of numerical and analytical methods.

The basic results of the dissertation work are in the following works:

1. Ахмедов, Н.К., Исмайылова, Д.Д. Напряженное состояние изотропного цилиндра с переменными модулями упругости // 1st international science and engineering conference. -Baku: Baku Engineering University, 29-30 november, -2018,- p.98-100.
2. Исмайылова, Д.Д. Анализ распространения осесимметричных упругих волн в радиально трехслойном цилиндре // «Riyaziyyatın tətbiqi məsələləri və yeni informasiya texnologiyaları» IV Respublika elmi konfransı, -Sumqayıt: Sumqayıt Dövlət Universiteti -09-10 dekabr, -2021, -s. 66-69.
3. Ismailova, J. Construction of homogeneous solutions for a radial inhomogeneous cylinder of small thickness // XXXIX Международная научно-практическая конференция, - Москва: -15 сентября -2021, -с.50-54.
4. Исмайылова, Д.Д. Задача кручения радиально-неодродного цилиндра // ВІСНИК Національного технічного університету "Харківський Політехнічний Інститут", -2017.(16), -с. 82-87.
5. Исмайылова, Д.Д. Анализ задачи кручения цилиндра с переменными модулями сдвига с закрепленной боковой поверхностью // - Баку: Ученые записки. Азербайджанский Технический Университет. -2017. № 1, -с. 88-93
6. Исмайылова, Д.Д. Крутильные колебания радиально-неоднородного изотропного цилиндра // -Баку: Ученые записки. Азербайджанский Технический Университет. - 2020, №1, -с. 42-48.
7. Ismailova, J. Studying elastic equilibrium of small thickness isotropic cylinder with variable elasticity module // - Baku: Transacti-

ons of NAS of Azerbaijan, issue Mechanics: -2019. 39(8), -p.17-23.

8. Akhmedov, N.K., Akbarova, S.B, Ismailova, J. Analysis of axisymmetric problem from the theory of elasticity for an isotropic cylinder of small thickness with alternating elasticity modules // Eastern-European Journal of Enterprise Technologies, -2019. 2/7 (98), -p.13-19.
9. Ismailova, J. Analysis of an axially-symmetric problem of elasticity theory for a radially-inhomogeneous cylinder mixed boundary conditions on lateral surfaces // -Baku: Transactions of NAS of Azerbaijan, issue Mechanics: -2021. 41(8), -p.30-38.

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